END

FILMED

DTIC
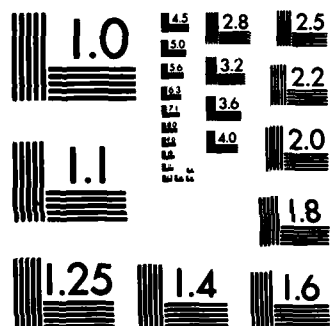
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

CRREL
REPORT 82-32

**US Army Corps
of Engineers**
Cold Regions Research &
Engineering Laboratory

Fluid dynamic analysis of volcanic tremor
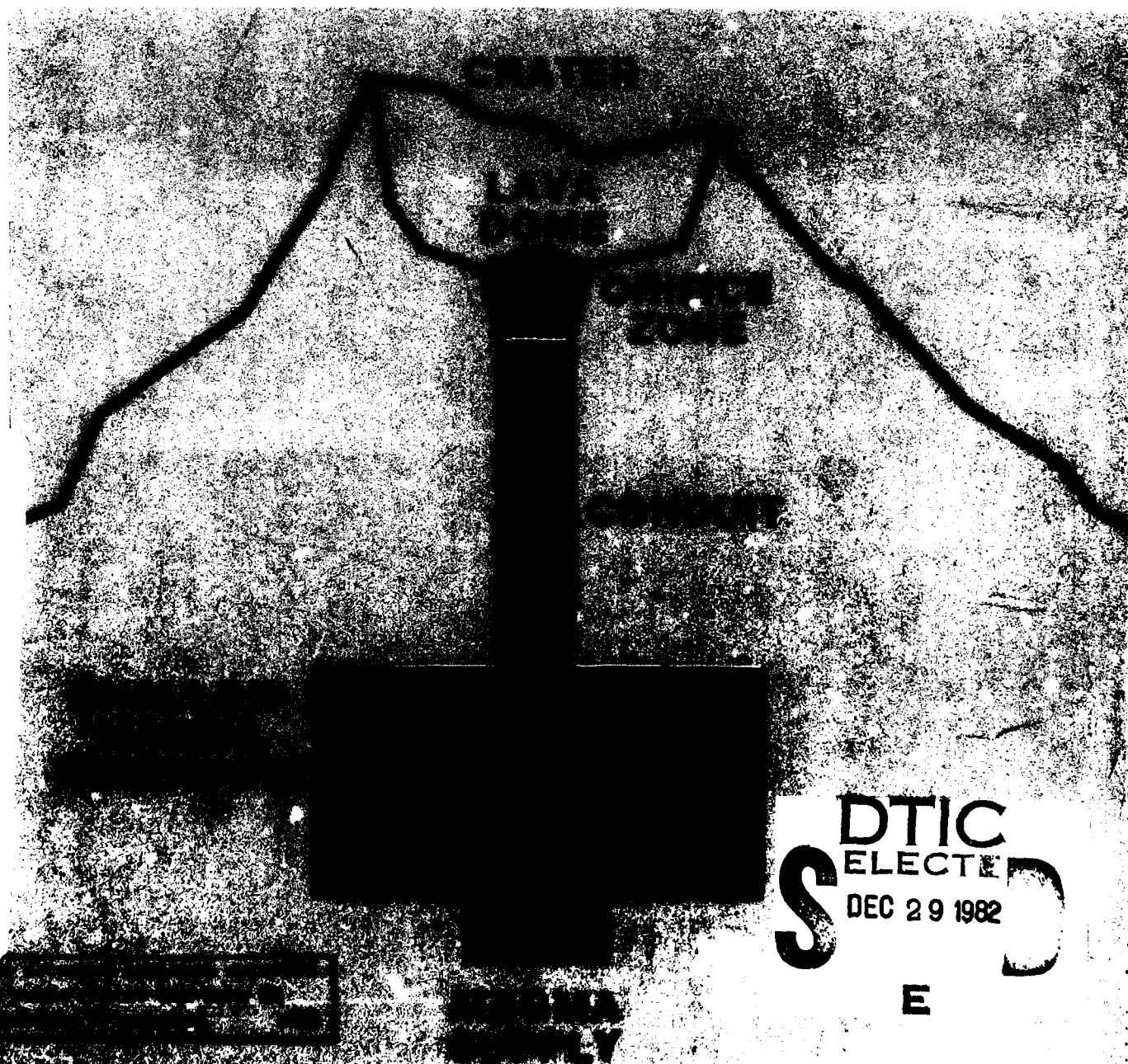
For conversion of SI metric units to U.S./ British customary units of measurement consult ASTM Standard E380, Metric Practice Guide, published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.

Cover: Idealized volcanic fluid system.

CRREL Report 82-32
October 1982

Fluid dynamic analysis of volcanic tremor

M.G. Ferrick, A. Qamar and W.F. St. Lawrence

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER CRREL Report 82-32	2. GOVT ACCESSION NO. AD-A122 778	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) FLUID DYNAMIC ANALYSIS OF VOLCANIC TREMOR		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) M.G. Ferrick, A. Qamar, W.F. St. Lawrence		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Cold Regions Research and Engineering Laboratory Hanover, New Hampshire 03755		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS DA Project 4A161102AT24 Task B, Work Unit 003
11. CONTROLLING OFFICE NAME AND ADDRESS Office of the Chief of Engineers Washington, D.C. 20314		12. REPORT DATE October 1982
		13. NUMBER OF PAGES 15
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Fluid dynamics
Seismology
Tremors
Volcanoes

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Low-frequency (< 10 Hz) volcanic earthquakes originate at a wide range of depths and occur before, during, and after magmatic eruptions. The characteristics of these earthquakes suggest that they are not typical tectonic events. Physically analogous processes occur in hydraulic fracturing of rock formations, low-frequency icequakes in temperate glaciers, and autoresonance in hydroelectric power stations. We propose that unsteady fluid flow in volcanic conduits is the common source mechanism of low-frequency volcanic earthquakes (tremor). The fluid dynamic source mechanism explains low-frequency earthquakes of arbitrary duration, magnitude, and depth of origin, as unsteady flow is independent of physical properties of the fluid and conduit. Fluid transients occur in both low-viscosity gases and

20. Abstract (cont'd)

high-viscosity liquids. A fluid transient analysis can be formulated as generally as is warranted by knowledge of the composition and physical properties of the fluid, material properties, geometry and roughness of the conduit, and boundary conditions. To demonstrate the analytical potential of the fluid dynamic theory, we consider a single-phase fluid, a melt of Mount Hood andesite at 1250°C, in which significant pressure and velocity variations occur only in the longitudinal direction. Further simplification of the conservation of mass and momentum equations presents an eigenvalue problem that is solved to determine the natural frequencies and associated damping of flow and pressure oscillations. For a simple constant pressure reservoir-conduit-orifice fluid system, a change in orifice size can cause the source signal duration due to a single disturbance to change from several seconds to in excess of an hour. Fluid kinematic viscosity ($0.412 \text{ m}^2/\text{s}$) is both included and neglected in the analysis to illustrate its effect upon system damping. Tremor magnitude is related to the magnitude of the conduit wall motion occurring in response to oscillating fluid pressures. The entire volcanic fluid system does not generally experience a transient flow condition at a given time. Closed end and constant pressure fluid system boundaries reflect pressure waves and isolate components of the fluid system. Total fluid movement may not be accurately monitored by tremor data alone, since steady state and slowly changing flows are aseismic.

PREFACE

Manuscript received 1 April 1981.

Accession For	
NTIS GRA&I	X
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
	Avail and/or
Dist	Special
A	

DTIC
COPY
INSPECTED
9

FLUID DYNAMIC ANALYSIS OF VOLCANIC TREMOR

M.G. Ferrick, A. Qamar and W.F. St. Lawrence

INTRODUCTION

The characteristics of volcanic earthquakes suggest that they are not typical tectonic events. Beginning with Omori (1914), many authors (e.g. Kubotera 1974, Minakami 1974a, Tanaka 1979) have described and classified volcanic earthquakes according to frequency content and duration of the seismic radiation. In general, investigators refer to three types of seismic signals produced by volcanic earthquakes that are not directly associated with eruptions: type A and B events and harmonic tremor. High-frequency (> 10-Hz) type A events have seismic signatures like those of tectonic earthquakes. Type

B events yield signals having a low-frequency content in the range of 0 to 5 Hz. Sustained low-frequency (0 to 10 Hz) ground motions lasting a few minutes to several hours are termed harmonic tremor. Figure 1 includes sample signatures of a type B event and a portion of a harmonic tremor.

Aki et al. (1977) have noted that low-frequency volcanic earthquakes have many characteristics that are similar to those observed in seismic signals emitted from the hydraulic fracturing experiments of Potter and Dennis (1974). Characteristics of volcanic tremor (Aki et al. 1977, Aki and Koyanagi 1981, Chouet 1981, Seidl et al. 1981) include a stationary seismic source, a peaked spectrum and a dominant frequency
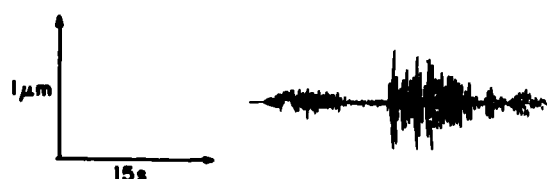
a. Type B volcanic earthquake (the large signal).

b. Two one-minute segments of a harmonic tremor recorded five minutes apart. This tremor lasted for about 20 minutes. The ground motion scale was calculated at a frequency of 1.5 Hz. In this figure a beating pattern can be noted.

Figure 1. Seismograms recorded at the Timberline Parking Lot on Mount St. Helens on 4 April 1980.

that can change during the course of eruptive activity.

The origin of low-frequency volcanic earthquakes is controversial. Some theories of the origin of the tremors are free oscillations of a magma chamber (Kubotera 1974, Sassa 1935, Shima 1958), oscillations of gas in a volcanic vent (Steinberg and Steinberg 1975), oscillation of volcanic layers from moving magma (Omer 1950) and random opening of tensile cracks from excess fluid pressure (Aki et al. 1977, Aki and Koyanagi 1981, Chouet 1981). All of these theories propose source effects as the cause of tremor. The observation of predominant low-frequency ground motion, even at epicentral distances of only a few kilometers, is evidence that the motion is produced at the source rather than being an effect of wave propagation through a heterogeneous medium. Seidl et al. (1981) noted the stability of low-frequency peaks obtained at the recording stations near Mount Etna and concluded that the origin was source related.

Volcanic earthquakes are known to occur at basaltic, andesitic and dacitic volcanoes as forerunners of eruptions (Minakami 1974b). Changes in seismic activity have, for example, been one of the consistent precursors of volcanic eruptions at Mount St. Helens, Washington. Over a thousand volcanic earthquakes exceeding magnitude 3, and at least 18 episodes of harmonic tremor, occurred before the major eruption of 18 May 1980. The most violent tremors occurred several hours after the initial explosion, and since that time the earthquakes have been much smaller. A marked increase in seismicity and one or more episodes of harmonic tremor have preceded and accompanied most subsequent eruptions (Decker and Decker 1981, Lipman and Mullineaux 1981).

In this analysis we seek an understanding of the source mechanism of both low-frequency harmonic tremor and volcanic (type B) earthquakes. Such an understanding allows a framework for interpretation of many apparently diverse observations. Physical analogies identify unsteady fluid dynamics as a likely source mechanism of tremor. Analyses of tremor to date have typically relied upon hypothesized fluid motions. Unlike previous investigations, we consider directly the dynamics of the fluid system. Equations describing the fluid vibration are developed from the physical laws of conservation of mass and momentum. Fluid properties characteristic of Mount St. Helens magmas are used to demonstrate the plausibility and analytical potential of the fluid dynamic source mechanism.

CONCEPT

Steady fluid flow, by definition, occurs with time-invariant fluid pressure at all points in the system. A fluid system at rest, for example, is at a steady (zero) flow. Excitations that occur slowly relative to the fundamental period of the fluid system induce gradual (aseismic) pressure variations. When a fluid system initially at steady state is abruptly disturbed, a period of transient flow results. All fluid systems, independent of conduit geometry and fluid properties, experience fluid transients when perturbed. Application of these concepts to volcanoes requires only that we presume the existence of a fluid system.

We propose that low-frequency volcanic events of various durations (volcanic tremor) have unsteady fluid flow as a common origin. The initial excitation to which the fluid system responds could be provided by a number of mechanisms. What is required is an abrupt flow input, an abrupt outflow, or some other perturbation of an existing steady condition. A likely mechanism, important during pre-eruptive periods, is the fracture of a conduit and the subsequent fluid response of a system initially at rest. Once initiated, flow and pressure oscillations continue until damped as a result of fluid friction, generation of seismic radiation and outflow of oscillatory energy from the system. The pressure oscillations in the fluid drive an oscillatory displacement of the conduit wall and generate elastic waves in the surrounding rock.

This source mechanism is suggested by a number of physical analogies. On 17 June 1976, the Edward Hyatt Power Plant at Oroville Dam, Oroville, California, experienced severe vibrations in penstock no. 1 (Ehrhart 1979). A transient flow condition in the penstock with severe pressure oscillations at a natural frequency of the fluid system was produced and maintained (forced) by a malfunction of the turbine shut-off valve. The low-frequency seismic signals emanating from the penstock were recorded by the network of seismometers in the area. Seismic activity ceased when the vibration was arrested. McNutt (1982) reported similar observations of low-frequency seismicity within 50 km of Tarbela Dam, Pakistan. He compared the seismic spectra near Tarbela Dam with the tremor spectra at six circum-Pacific stratovolcanoes and found them to be similar, each exhibiting narrow, evenly spaced peaks. An important observation addressed analytically in this paper was that the recorded spectra near Tarbela Dam changed with the size of the outflow opening of the tunnel.

St. Lawrence and Qamar (1979a, 1979b) observed that low frequency glacial icequakes and volcanic
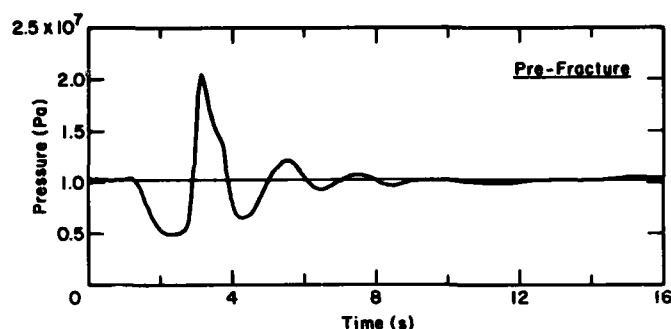
a. Before hydraulic fracture treatment.
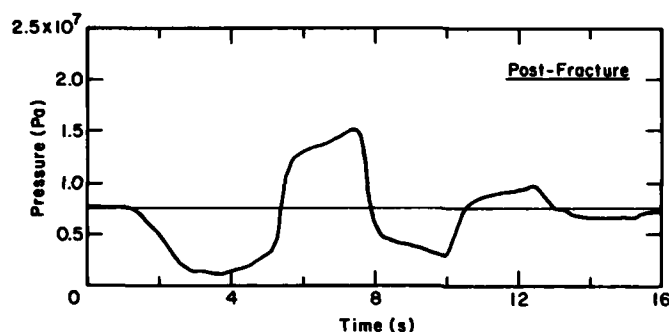
b. After hydraulic fracture treatment.

*Figure 2. Time variation of fluid pressure near the surface of a
well of 1590-m depth and 0.062-m diameter. The fluid transient
was induced by rapid fluid withdrawal at the surface in each in-
stance. The frequency of the oscillation has decreased by more
than a factor of two between these pulse tests. Static pressure
is greatly exceeded during each transient flow condition.*

earthquakes produce seismic signals that cannot be
differentiated from one another. A common link
between these two physical systems is that each in-
cludes fluid-filled conduits.

Anderson and Stahl (1967) conducted field tests
to investigate the process of hydraulic fracturing.
Using formation crude oil as the fracture fluid, they
observed fluid pressure oscillations in the wellbore.
Subsurface fluid pressure transients generated during
hydraulic fracturing of the formation exhibited the
same characteristics both deep in the well and near
the surface. Fracturing of the formation caused the
period of the oscillations to increase.

Recently we conducted experiments in conjunc-
tion with a hydraulic fracture treatment of a rock
formation near the bottom of a 1590-m well of
0.062-m diameter. The fluid that was used had a
relatively high viscosity of about 80 centipoise, which
together with the small diameter conduit gave a

highly damped response. Testing included pre-fracture
and post-fracture "pulse" tests in which a small volume
of fluid was abruptly released, by valving at the surface,
from the pressurized fluid system which was initially
at rest. Fluid pressure variations with time at a point
near the surface, as a result of the fluid release, are
presented in Figure 2 for pre- and post-fracture tests.

The pre-fracture fluid system consisted of a con-
duit which was closed at both ends so that zero flow
boundaries were present. The pressure-time trace
for this case exhibits a dominant frequency of 0.44
Hz. Increasing pressure near the surface is accompan-
ied by fluid flow toward the surface and conversely
for decreasing pressure.

After the fracture treatment of the formation, a
large volume of fluid remained in the system of frac-
tures which intersected the well near its base. The
post-fracture pulse test yielded a dominant frequency
of 0.17 Hz. The downhole boundary had changed

3

from a zero flow boundary to a constant pressure "reservoir" boundary. This change results in a doubling of the fundamental period of the fluid system. The post-fracture period was greater than twice the pre-fracture period, indicating that the conduit length has increased. The post-fracture trace exhibts line packing, a characteristic of high friction fluid systems. Line packing occurs when flow is not totally stopped by the passage of the compression wave; the pressure near the closed end of the pipe continues to rise for a period of time following the abrupt pressure increase. In each pulse test, fluid pressures recorded near the surface during the transient flow condition greatly exceeded the existing static pressure.

Seidl et al. (1981), following St. Lawrence and Qamar (1979a, 1979b), have applied basic concepts of the fluid transient theory to the analysis of tremor at Mount Etna. Analysis of seismometer records near the volcano revealed that eruptions can be distinguished according to different eigenfrequencies of the fissure and conduit network. Positions of active dykes and conduits were traced following the amplitude distributions associated with pronounced spectral lines in the tremor spectra. In addition, it was found that harmonic tremor and volcanic (type B) earthquakes have similar spectra.

LeGuern et al. (1982) observed several periodic swarms of between 10 and 14 emissions of gas from a vent on Mount Etna. Each swarm had a period of 1.8 s, decreased in intensity with each succeeding emission in the swarm, and was typically followed by a pause of a few minutes. Flow velocity, dynamic pressure and temperature measured at the vent revealed that the phenomenon was due to resonance of the volcanic gas in the conduit.

BACKGROUND

Pressure and flow oscillations occur in fluid systems at a frequency that depends upon both the geometry of the container and the location of the excitation (Muto and Kanei 1980, Zielke and Hack 1972). The excitation need only be temporary to induce a fluid oscillation.

Analysis of the fluid system of a specific volcano may require that a multiphase fluid (liquid, gas, solid in suspension) having significant temperature variations be considered. The general theory is complex and full exposition is unwarranted in this discussion. Therefore, we will consider a single phase fluid (liquid) in which temperature variability does not significantly affect the flow. The flow of a gas could also be considered with relatively minor modifications of this development (Wylie and Streeter 1978). The

nature of a fluid transient in a closed conduit is then dependent upon fluid density, viscosity and pressure wave speed, length and shape of the conduit, boundary conditions and the nature of the disturbance.

The physical laws that describe the flow of a liquid in a conduit are (Wylie and Streeter 1978) the conservation of momentum

$$g \frac{\partial H}{\partial x} + \frac{Q}{A^2} \frac{\partial Q}{\partial x} + \frac{1}{A} \frac{\partial Q}{\partial t} + F(Q) = 0 \qquad (1)$$

and the conservation of mass

$$\frac{\partial H}{\partial t} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} + \frac{Q}{A} \frac{\partial H}{\partial x} - \frac{Q}{A} \sin \alpha = 0 \qquad (2)$$

where

H = the pressure head in meters,
Q = the volumetric flow rate (m^3/s),
A = the cross-sectional area of the conduit (m^2),
$F(Q)$ = a frictional loss function which is dependent upon fluid viscosity, conduit size, shape, and roughness,
α = the angle of the conduit with the horizontal,
x = linear distance along the conduit,
g = the acceleration due to gravity,
a = the pressure wave speed in the field,
t = time.

These quasi-linear hyperbolic equations together with appropriate boundary and initial conditions can be solved numerically. The governing equations as stated require the assumption that the conduit material behaves elastically. The generation of seismic radiation is accomplished without energy loss by the fluid system. If the elastic assumption cannot be made, a stress-strain model describing conduit material behavior is required and the corresponding fluid dynamic model would exhibit increased damping.

For hyperbolic equations, H and Q information propagates in the x-t plane along characteristics. The slope of a characteristic in closed conduit flow applications is dominated by the pressure wave speed in the fluid:

$$\frac{dx}{dt} = \frac{Q}{A} \pm a \approx \pm a. \qquad (3)$$

Both flow and pressure at all points in the fluid respond rapidly to a disturbance in the system.

Equations 1 and 2 are typically the basis of analyses of transients in fluid systems for which the system geometry, boundary conditions and fluid properties are known. Given sufficient data at a specific volcano, the behavior of the fluid transient source

4

could be modeled in a precise way by using these or more general equations. Typically, however, the configuration of the fluid system is poorly known. We are then faced with the inverse problem of finding the fluid system geometry and the precise nature of the frictional loss function. The solution of this problem is not unique. Volcanoes with different magmas and conduit system geometries could generate similar low-frequency seismic signals.

As steady fluid flow is aseismic, it usually is not possible to quantify the total volume of magma flow using seismic data generated during episodes of volcanic tremor. If low-frequency pressure oscillations at the source can be determined with reasonable confidence, however, fluid dynamic modeling tools can be applied to investigate both the rate of oscillatory fluid flow and the physical characteristics of the fluid system.

LINEAR THEORY

The basic features of a fluid system response to a disturbance, including the natural frequencies of the vibration and associated damping of the oscillations, can be conveniently determined with a free vibration analysis (Wylie and Streeter 1978, Zielke and Hack 1972). The nonlinear terms, $Q/A^2 \, \partial Q/\partial x$ and $Q/A \, \partial H/\partial x$ in eq 1 and 2 respectively, are disregarded in this analysis as fluid velocity and the spatial gradients of head and discharge are typically small. The frictional loss term, $F(Q)$, which is non-linear for turbulent flow, must be linearized. Also, for cases where the volumetric flow rate is not extremely large, the $\sin \alpha$ term in the conservation of mass equation is small and can be neglected. Finally, the dependent variables, H and Q, are decomposed into mean $(\overline{H}, \overline{Q})$ and fluctuating (h', q') components.

The linearized equations for oscillatory flow are then

$$\frac{\partial h'}{\partial x} + \frac{1}{gA} \frac{\partial q'}{\partial t} + Rq' = 0 \qquad (4)$$

$$\frac{\partial q'}{\partial x} + \frac{gA}{a^2} \frac{\partial h'}{\partial t} = 0 \qquad (5)$$

where R is the linearized resistance per unit length. These equations can be cast in the form of the telegraph (wave) equation for both h' and q':

$$\frac{\partial^2 q'}{\partial x^2} = \frac{1}{a^2} \frac{\partial^2 q'}{\partial t^2} + \frac{gAR}{a^2} \frac{\partial q'}{\partial t} \qquad (6)$$

$$\frac{\partial^2 h'}{\partial x^2} = \frac{1}{a^2} \frac{\partial^2 h'}{\partial t^2} + \frac{gAR}{a^2} \frac{\partial h'}{\partial t} \qquad (7)$$

With the assumption that $h' = X(x)T(t)$, the method of separation of variables can be applied to eq 7 yielding

$$\frac{1}{X} \frac{d^2X}{dx^2} = \frac{1}{T}\left(\frac{1}{a^2} \frac{d^2T}{dt^2} + \frac{gAR}{a^2} \frac{dT}{dt}\right) = \gamma^2 \qquad (8)$$

where γ^2 is the separation constant. The particular solution for an oscillatory pressure head can be found from eq 8 as

$$h' = e^{st}\left(C_1 e^{\gamma x} + C_2 e^{-\gamma x}\right)$$
$$\gamma^2 = \frac{s}{a^2}(s + gAR) \qquad (9)$$

where C_1 and C_2 are constants.

The complex frequency s is a constant composed of a real part σ which quantifies system damping, and an imaginary part ω the circular frequency of the oscillation. Substituting eq 9 into eq 4 and 5 yields the solution for q'

$$q' = \frac{-gA}{a^2} \frac{s}{\gamma} e^{st}\left(C_1 e^{\gamma x} - C_2 e^{-\gamma x}\right) \qquad (10)$$

If the upstream end of the conduit is identified with subscript U, the constants C_1 and C_2 are determined as

$$C_1 = \frac{1}{2}(H_u - Z_c Q_u)$$
$$C_2 = \frac{1}{2}(H_u + Z_c Q_u)$$
$$Z_c = \frac{a^2}{gA} \frac{\gamma}{s}. \qquad (11)$$

The equations for h' and q' can then be written

$$h' = (H_u \cosh \gamma x - Z_c Q_u \sinh \gamma x) = H(x) e^{st}$$
$$q' = -\frac{H_u}{Z_c} \sinh \gamma x + Q_u \cosh \gamma x = Q(x) e^{st}. \qquad (12)$$

The expressions for $H(x)$ and $Q(x)$ are termed transfer equations for head and discharge and can be used to relate H and Q at each end of a conduit.

To analyze a given fluid system, transfer equations for all lines and additional equations for terminal conditions, conduit branch points and other components are assembled. The resulting system of transfer equations is then written as

5

$$M_s x = 0 \qquad (13)$$

where M_s is the overall system matrix, and x is a vector of pressures and flows. The values of s for which the matrix equation has a nontrivial solution are those for which the determinant of M_s vanishes. The residual time response of a fluid system that has undergone an excitation is known when the complex frequency is found by solution of this eigenvalue problem (Zielke and Hack 1972). Because the fluid system is continuous, an infinite number of complex frequencies can be found. Practically, however, only the lowest few harmonics are observed.

A consequence of the linear theory is that unsustained disturbances in fluid systems decay exponentially with time (eq 12). Envelopes of dimensionless amplitude decay are plotted in Figure 3 as a function of time. Depending upon the magnitude of σ, oscillations can persist for a few seconds or in excess of an hour as the result of a single disturbance.

Two parameters of this analysis are important for characterizing the response of the fluid system of the volcano. The characteristic impedance, Z_c, is a function of fluid and conduit properties. In large diameter pipes, the value of the characteristic impedance approaches that for frictionless sytems (eq 9 and 11):

$$Z_c = \frac{a}{gA} . \qquad (14)$$

The fracture of the pipe wall becomes an orifice through which fluid flows out of the system. The orifice impedance Z_{or} is defined as

$$Z_{or} = \frac{2\overline{H}}{\overline{Q}} \qquad (15)$$

where $\overline{H}$ is the mean pressure head and $\overline{Q}$ is the mean flow just upstream of the orifice. A relatively large opening in the conduit through which fluid passes easily with only a small loss of pressure head has a small orifice impedance. A small conduit opening which does not readily pass fluid has a large orifice impedance.

The frequency and damping characteristics of the fluid system are greatly influenced by the relative magnitudes of the characteristic and orifice impedances. For a frictionless reservoir-conduit-orifice fluid system, $H_u = 0$ and $R = 0$. Identifying the downstream end of the conduit, near the orifice, with subscript D,

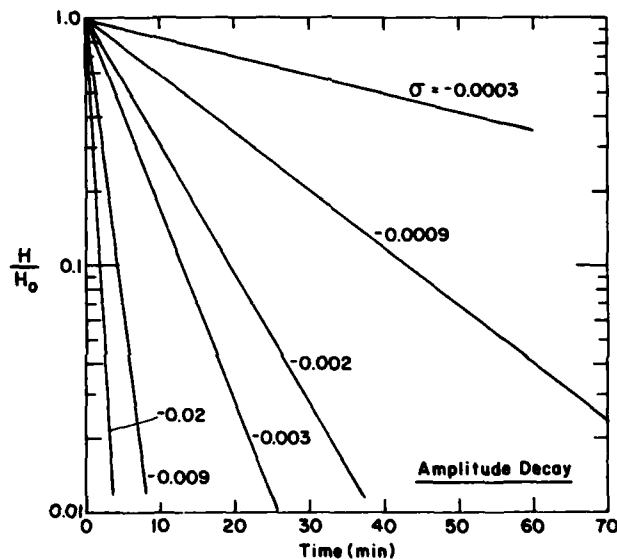$$Z_D \equiv \frac{H_D}{Q_D} = Z_{or} . \qquad (16)$$

Equations 12, 14–15 then yield

$$Z_{or} \cosh \gamma l + Z_c \sinh \gamma l = 0 \qquad (17)$$

where l is the conduit length. Solution of the imaginary part of eq 17 gives the circular frequencies
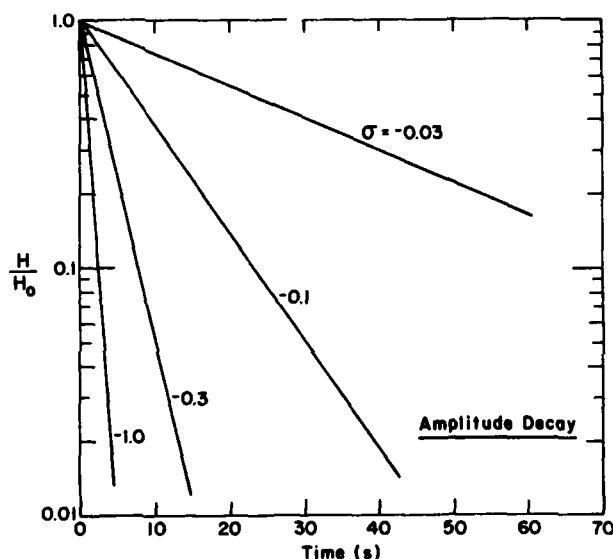
a.

b.

Figure 3. Envelopes of dimensionless amplitude decay of an oscillation for a range of system damping values (σ). H_0 is the original or maximum amplitude of the oscillation. Note the change of scale of the time axis between Figures 3a and 3b.
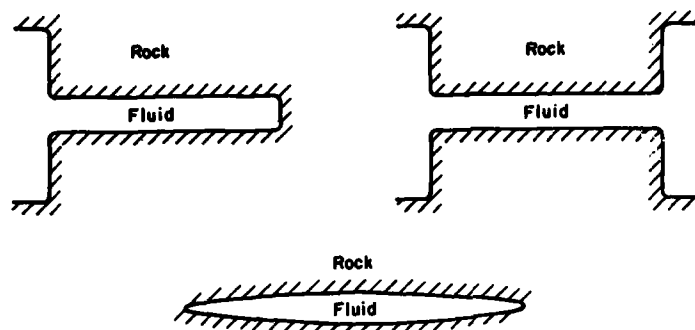
6

Figure 4. Single conduit components of a fluid system. Intersections with a significantly larger conduit, a large volume fracture zone or a closed end are boundary conditions that isolate components of the fluid system.

of the system and the real part of eq 17 is solved to obtain system damping. When $Z_{or} > Z_c$, the fluid system responds as if the conduit were a closed pipe and the frequencies are the odd harmonics of the conduit:

$$\omega = 2\pi f = \frac{n\pi a}{2L}, n = 1,3,5,...$$

$$\sigma = \frac{a}{2L} \ln \frac{Z_{or} - Z_c}{Z_c + Z_{or}}. \tag{18}$$

If $Z_c > Z_{or}$, the system responds as if the conduit were open-ended and the frequencies are the even harmonics of the conduit:

$$\omega = 2\pi f = \frac{n\pi a}{2L}, n = 2,4,6...$$

$$\sigma = \frac{a}{2L} \ln \frac{Z_c - Z_{or}}{Z_c + Z_{or}}. \tag{19}$$

For both cases, system damping increases as the magnitudes of the characteristic and orifice impedance approach each other. Less pressure wave reflection occurs at the orifice and oscillatory energy passing out of the system increases. As the ratio of the magnitudes departs from one, more energy is reflected and system damping decreases.

The dominant range of tremor frequencies observed at a number of volcanoes is 1 to 2 Hz. The natural frequencies of a fluid transient are a function of the pressure wave speed in the fluid and of conduit length and boundary conditions (eq 18). Much lower frequencies would be expected if the complete fluid system within the volcano were experiencing a tran-

sient condition at a given time. A segment of the system is fluid-dynamically isolated from the remainder of the fluid system when the boundary condition provides total pressure wave reflection. This can occur physically by conduit pinchout given a closed end boundary condition, or by intersection with a significantly larger conduit or a large volume fracture zone creating a reservoir or constant pressure boundary condition (Fig. 4). The complete volcanic fluid system is visualized as a tree-like system of these components. Any component of the system can experience a transient flow-induced vibration.

APPLICATION OF LINEAR THEORY: MOUNT ST. HELENS

To demonstrate the potential insights that can be obtained from the fluid dynamic theory, we will now make an assessment of some features of volcanic tremor at Mount St. Helens. The fluid system segment to be analyzed is assumed to be composed of a relatively large body of magma connected to a smaller conduit which terminates in a closed end. The larger magma body could be a chamber, a large conduit, or an extensive zone of fractures containing a volume of magma that is large relative to the volume contained in the conduit itself. The smaller conduit is assumed to be nearly vertical, with an approximately circular, prismatic cross section and a diameter of 100 m. The effect of choosing a relatively large diameter conduit is to decrease system damping due to fluid friction. The same fluid in a smaller conduit would exhibit greater viscous damping. The cylindrical conduit shape is chosen to take full advantage of existing fluid transient theory. A relatively

7

short conduit length is indicated by the observed tremor frequencies (Fig. 1). An "order of magnitude" conduit length of 1000 m is chosen for this analysis.

Murase and McBirney (1973) experimentally determined the fluid density ρ, kinematic viscosity v, bulk modulus K, and pressure wave speed of melts of Mount Hood andesite composed of 60% SiO_2. The melt was totally fluid at temperatures greater than 1200°C. For an assumed temperature of 1250°C, the parameter values were given as $\rho = 2430$ kg/m^3, $v = 0.412$ m^2/s, $K = 1.78 \times 10^{10}$ Pa, and the wave speed in an unbounded fluid, $a = 2700$ m/s. It is reasonable to expect that the degassed melt had a much higher viscosity than in-situ magma at the same temperature. The decreased viscous damping due to the lower in-situ viscosity is neglected in this analysis.

In closed conduit flow, conduit properties affect the pressure wave speed of the fluid. If we again assume that the conduit is in an elastic medium and is constrained from longitudinal movement, the wave speed can be expressed as (Wylie and Streeter 1978):

$$a^2 = \frac{K/\rho}{1 + \frac{2K}{E_R}(1+\mu)} \qquad (20)$$

where μ and E_R are, respectively, Poisson's ratio and the modulus of rigidity of the conduit. Shear and pressure wave speeds in cool Mount Hood andesite are given by Murase and McBirney (1973) as 2700 and 5500 m/s, respectively, and density is given as 2500 kg/m^3. The modulus of rigidity and Poisson's ratio of the conduit material are then estimated as 1.77×10^{10} Pa and 0.34 respectively. Other factors, not considered in eq 20, may influence the fluid wave speed. Jenkner (1971) has shown that the wave speed of a fluid in a rectangular conduit is reduced significantly relative to that in a circular conduit. Also, the effect of trapped gas volumes or entrained gas bubbles upon wave speed may be important if degassing of magma is suspected in low pressure regions of the fluid.

Initially the system is pressurized, but the fluid is at rest. As a result of processes occurring at depth, additional fluid gradually moves into the system, and the static pressure increases. Finally the increasing pressure causes stresses to develop in the rock wall of the conduit which exceed the strength of the conduit, resulting in a fracture near the closed end and an abrupt outflow of fluid. The response of the fluid in the conduit due to this outflow is of interest.

Neglecting friction, the characteristic impedance of the assumed segment of the Mount St. Helens fluid system is 0.0183 using eq 14 and 20. The variation of fluid system damping σ with orifice impedance is plotted in Figure 5. By using Figure 3 to translate system damping to decay time of a disturbance, it is clear that even in the absence of friction, dissipation of oscillatory energy via transmission out of the system can be large.

A numerical solution of the free vibrational eigenvalue problem was obtained to determine the complex frequencies of the fluid system subject to the frictional effects of the high viscosity melt. Circular frequencies were unchanged from the values given in eq 18 and 19. The lowest five harmonics have periods ranging from 0.6 to 2.8 s, consistent with our observations of low-frequency oscillations at Mount St. Helens.

The magnitude of system damping σ, including the effects of friction, was the same for the first, third and fifth harmonics for the single conduit fluid system. The same was true for the even harmonics of the simple fluid system. The variation of damping with orifice impedance is again given in Figure 5. As the orifice impedance approaches the characteristic impedance, the high viscosity and frictionless plots coincide. This condition is termed a matched line. In this case, the majority of the energy in the oscillation is not being reflected at the orifice, and the damping of the oscillation due to the transmission of energy from the system dominates the effects of friction. At the highest and lowest flow extremes, where the ratio of the impedances is much different from one, most of the energy in the oscillation is being reflected by the conduit termination. The effect of friction upon system damping becomes apparent, and a lower limit on damping is indicated. For a chosen value of orifice impedance, comparison of the damping values given by the two curves indicates the importance of viscous damping upon the overall damping of an oscillation.

Figure 5 is symmetric about the matched line condition, $Z_c = Z_{or}$. Fluid transients provide a common source mechanism for both pre-eruptive tremor and tremor occurring during eruptions. Odd harmonics of the conduit, corresponding to a closed pipe condition, occur when orifice impedance is larger than the characteristic impedance of the system. During pre-eruptive activity, the conduit behaves as a closed pipe. The fluid outflow rate is relatively low and orifice impedance is large. Tremors of various durations, represented by the right half of Figure 5, may accompany fluid movement. The left half of Figure 5, where orifice impedance is low, corresponds to a relatively large volumetric outflow through an open-ended conduit. Even harmonics of the conduit occur during these open-ended, low orifice impedance conditions. Again, tremors of various durations can be expected.
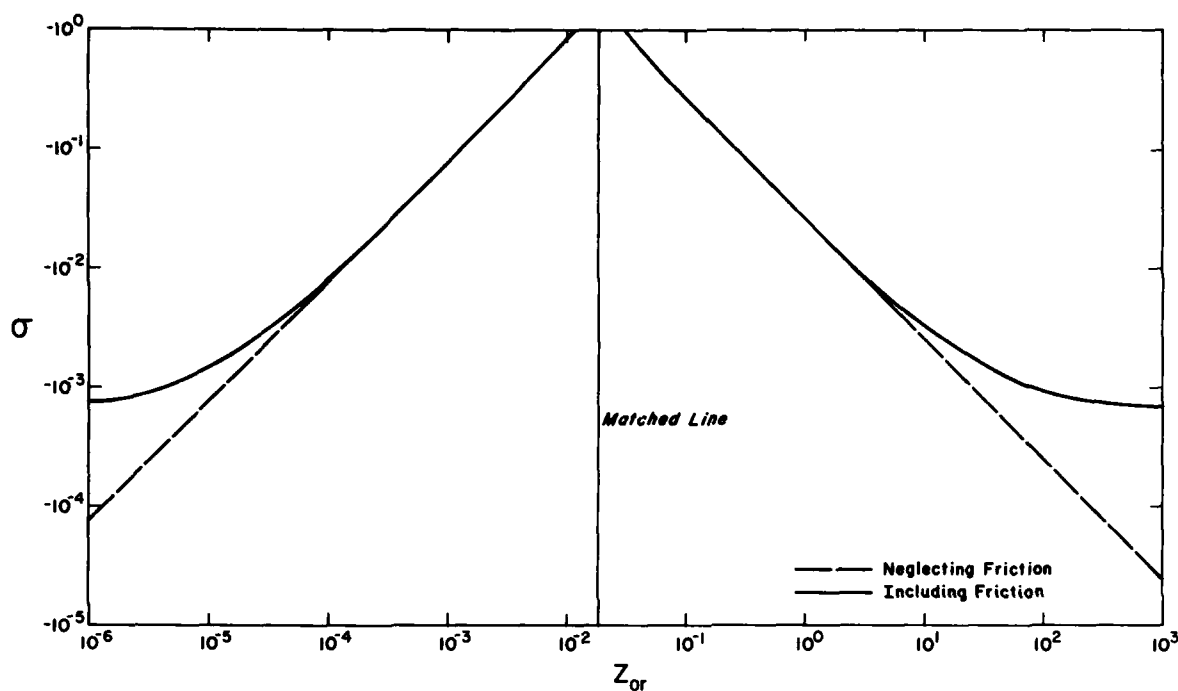
8

Figure 5. Variation of system damping for the single conduit fluid system and assumed parameter values for Mount St. Helens, as a function of orifice impedance. Values of σ are plotted both neglecting and including fluid frictional effects. Small orifice impedance values correspond to relatively large outflow rates, while large values reflect relatively small flow rates. For a matched line condition, $Z_c = Z_{or}$, damping is very large.

The current period of seismic activity at Mount St. Helens began in March 1980. Since that time, episodes of volcanic tremor with durations ranging from several seconds to more than an hour have been recorded. As depicted in Figure 5, the unsteady flow source mechanism reveals the potential for a continuous variation of system damping as a function of the hydraulic characteristics of the orifice or fracture. As orifice conditions change, low-frequency earthquake durations resulting from a single disturbance can potentially vary between the shortest duration type B event and the lengthy episodes of harmonic tremor recorded. Large differences in the durations of these signals mask the existence of a common source mechanism. If outflow conditions are not changing significantly and short duration type B events have recently been recorded, subsequent tremor originating from the same location would likely be due to either a number of excitations of the fluid system that are closely spaced in time or the existence of some mechanism that is forcing the oscillation.

The many physical changes that have occurred in the vicinity of the present lava dome at Mount St. Helens potentially allow a wide variation of tremor durations as a result of an excitation of the near-surface portion of the fluid system. The fluid dynamic source mechanism could be responsible for low- frequency vibration in any segment of the volcanic fluid system. Recent dome building events and eruptions, however, are signaled only by near-surface fluid transients centered near the surface outlet.

The amplitude of volcanic tremor at Mount St. Helens has varied over several orders of magnitude. In general, tremors of smaller magnitude have been recorded since the catastrophic eruption of 18 May 1980. The amplitude of a tremor is proportional to the amplitude of the pressure oscillation in the conduit. Amplitudes that vary over a wide range are possible both during and prior to an eruption.

Prior to an eruption, a limiting condition on tremor amplitude is provided by the overburden pressure at the point of fluid outflow. Larger tremor amplitudes are possible before catastropic failure if a large overburden is present. Pressures greatly exceed static pressure during unsteady flow events. Rock that fractures to initiate the unsteady flow is likely to fracture further during the transient.

During the period of 23 April–17 May 1980, the north flank bulge at Mount St. Helens expanded at a

rate of approximately 1.5 m per day. As the area of the bulge was about 4 km^2, this expansion rate corresponds to an average continuous outflow of approximately 70 m^3/s from the conduit. Depending upon orifice impedance, oscillatory flows of less than ± 100 m^3/s could produce initial pressure oscillations of $\pm 2.4 \times 10^6$ Pa. Much larger pressure oscillations prior to 18 May are not probable as the order of magnitude of the overburden pressure of the mountain would have been approached. Relieved of the massive overburden, static pressure and tremor amplitudes are limited. Magmatic eruptions should continue to yield relatively smaller volumes of material until a large overburden pressure is again developed.

A characteristic of Mount St. Helens similar to those of Hawaiian volcanoes is the presence of tremors during major volcanic activity (Kinoshita et al. 1969). Tremor amplitudes in both cases typically subside as the intensity of the activity diminishes. As an eruption wanes and mean flow diminishes, the magnitude of flow oscillations can also be expected to diminish. For an orifice that is not radically changing in size, orifice impedance is constant. As pressure and flow oscillations at the orifice are related by the orifice impedance (eq 16), the magnitude of pressure oscillations at the orifice diminishes with the flow. If we assume that the second harmonic of the conduit is excited, the maximum pressure oscillation occurs at the mid-length of the conduit. Its magnitude is related directly to that at the orifice by the transfer functions. It is generally believed that more than 1 km^3 of material was ejected from Mount St. Helens on 18 May 1980. The greatest portion of the flow occurred during the first 9 hours of the eruption. The average flow rate during that period can then be estimated at 30,000 m^3/s. Small pressure oscillations at the orifice, corresponding to a flow oscillation of $\pm 5,000$ m^3/s, yield a pressure oscillation of $\pm 2.4 \times 10^6$ Pa at the pipe mid-length. As the outflow oscillations diminish by an order of magnitude, the maximum pressure oscillations diminish correspondingly.

For an elastic pipe, radial wall deformation in response to the oscillatory pressure can be estimated. These calculations suggest that sufficient energy is available via the fluid dynamic source mechanism to generate the range of observed tremor magnitudes.

Preliminary examination of our seismograph records indicates a variation in the spectra of both type B events and harmonic tremor episodes. One explanation is that different harmonics are excited at different times. Other possibilities are that the outflow location has changed, modifying the effective conduit length, or that the conduit length has increased due to a hydraulic fracturing. Another possibility is that different components of the fluid system are experiencing transient conditions at different times.

Many low-frequency seismic signals generated at volcanoes exhibit an emergent onset. Over the first few cycles, the magnitude of the oscillation increases. If this characteristic is source-related, it requires a modification of the static reservoir-conduit-orifice fluid system component. In general, fluid transients generate the largest pressure oscillations just after the disturbance occurs. Without continued forcing, these oscillations decay in time. A diaphragm inserted upstream of the orifice, to account for conduit volume change in response to fluid pressure change, will produce an emergent fluid pressure oscillation. An orifice that is changing in size with time could also produce an emergent pressure oscillation.

Beating is another characteristic observed in the low-frequency seismic output of volcanoes (Fig. 1). The combined effects of different but concurrent outflows from a segment of the fluid system will produce beating oscillatory pressures in the fluid. Alternatively, different segments of the fluid system could be experiencing transient flow conditions concurrently. This concurrent transient flow condition occurs, for example, when two conduits of unequal length are connected by a crack which opens to allow fluid passage when a sufficient pressure difference across it develops.

CONCLUSIONS

We have investigated the concept of a fluid dynamic source mechanism of low-frequency volcanic earthquakes. The concept is suggested by analogous physical processes occurring in three other settings: 1) hydraulic fracturing of rock formations, 2) low frequency icequakes in glaciers and 3) auto-resonance in hydropower stations.

The analysis has linked long and short duration tremors originating at arbitrary depth, occurring both prior to and during volcanic eruptions, to a common source mechanism. The existence of tremors of fluid dynamic origin is independent of fluid properties and conduit geometry, although the nature of the tremor is affected by these parameters.

The fluid dynamic source mechanism exhibits all of the features generally attributed to the source mechanism of volcanic tremor including 1) a stationary seismic source, 2) a source location that can vary between tremor episodes, 3) a peaked frequency spectrum and 4) a dominant frequency which can change during the course of eruptive activity.

The fluid dynamic theory can be applied in as much generality as is warranted by data availability. We have used features of the recent eruptive sequence at Mount St. Helens to demonstrate the application of the linear theory of unsteady fluid flow. With more precise information regarding the fluid system configuration, tremor magnitudes and source locations, greater quantitative insight could be obtained. Even without this information, the analysis has provided several insights. Variable system damping can result in a broad range of low-frequency seismic signal durations. For a simple system, damping is affected by characteristics of the fluid, conduit and outflow. It has been demonstrated that, for a given system, a change in outflow conditions is all that is required for large changes in system damping characteristics.

The magnitudes of tremors are related directly to the magnitude of oscillatory fluid pressures. For the case of pre-eruption tremor, these pressures are an indication of the magnitude of the potential eruption. Tremor amplitudes during an eruption generally monitor its progress, decreasing as the eruption wanes. Total fluid movement cannot be quantified from volcanic tremor data alone since steady-state or slowly changing fluid flows are aseismic.

LITERATURE CITED

Aki, K., M. Fehler and S. Das (1977) Source mechanism of volcanic tremor: Fluid-driven crack models and their application to the 1963 Kilauea eruption. *Journal of Volcanology and Geothermal Research*, vol. 2, p. 259-287.

Aki, K. and R. Koyanagi (1981) Deep volcanic tremor and magma ascent mechanism under Kilauea, Hawaii. *Journal of Geophysical Research*, vol. 86, p. 7095-7109.

Anderson, T.O. and E.J. Stahl (1967) A study of induced fracturing using an instrumental approach. *Journal of Petroleum Technology*, vol. 19, p. 261-267.

Chouet, B. (1981) Ground motion in the near field of a fluid-driven crack and its interpretation in the study of shallow volcanic tremor. *Journal of Geophysical Research*, vol. 86, p. 5985-6016.

Decker, R. and B. Decker (1981) The eruptions of Mount St. Helens. *Scientific American*, vol. 244, p. 68-80.

Ehrhart, R.W. (1979) Auto-oscillation in the Hyatt plant's penstock system. *Water Power and Dam Construction*, vol. 31, p. 38-41.

Jenkner, W.R. (1971) Uber die Druckstrossgeschwindigkeit in Rohrleitungen mit quadratischen und rechteckigen Querschnitten. *Schweizerische Bauzeitung*, vol. 89, p. 99-103.

Kinoshita, W.T., R.Y. Koyanagi, T.L. Wright and R.S. Fiske (1969) Kilauea volcano: The 1967-68 summit eruption. *Science*, vol. 166, p. 459-468.

Kubotera, A. (1974) Volcanic tremors at Aso volcano. In *Developments in Solid Earth Geophysics*, Amsterdam: Elsevier, p. 29-47.

LeGuern, F., H. Jazieff, C. Vavasseur, and P. Zettwoog (1982) Resonance in the gas discharge of the Bocca Nuova, Etna (Italy), 1968-1969. *Journal of Volcanology and Geothermal Research*, vol. 12, p. 161-166.

Lipman, P.W. and D.R. Mullineaux, eds. (1981) 1980 eruptions of Mount St. Helens, Washington. Geological Survey Professional Paper 1250.

McNutt, S. (1982) Analysis of volcanic tremor from Pavlof, Fuego, Pacaya, San Cristobal, Massaya, and Poaz Volcanoes. *EOS, Transactions of the American Geophysical Union*, vol. 63, p. 459.

Minakami, T. (1974a) Seismology of volcanoes in Japan. In *Developments in Solid Earth Geophysics*, Amsterdam: Elsevier, p. 1-27.

Minakami, T. (1974b) Prediction of volcanic eruptions. In *Developments in Solid Earth Geophysics*, Amsterdam: Elsevier, p. 313-333.

Murase, T. and A.R. McBirney (1973) Properties of some common igneous rocks and their melts at high temperatures. *Geological Society of America, Bulletin*, vol. 84, p. 3563-3592.

Muto, T. and T. Kanei (1980) Resonance and transient response of pressurized complex pipe systems. *Bulletin, Japanese Society of Mechanical Engineers*, vol. 23, p. 1610-1617.

Omer, G.C. (1950) Volcanic tremor. *Bulletin, Seismological Society of America*, vol. 40, p. 175-194.

Omori, F. (1912-1914) The eruptions and earthquakes of the Asamayama. *Bulletin, Imperial Earthquake Investigation Commission*, Tokyo, vol. 6, p. 257 (in Japanese)

Potter, R.M. and B.R. Dennis (1974) Seismic and fluid pressure response from a series of hydraulic fractures in granite. *EOS, Transactions of the American Geophysical Union*, vol. 55, p. 430.

Sassa, K. (1935) Volcanic micro-tremor and eruption earthquakes. Memorandum of the College of Science: Kyoto University, vol. 18, p. 255-293.

Seidl, D., R. Schick and M. Riuscetti (1981) Volcanic tremors at Etna: A model for hydraulic origin. *Bulletin of Volcanology*, vol. 44, p. 43-56.

Shima, M. (1958) On the second volcanic microtremor at Volcano Aso. Disaster Prevention Research Institute, Kyoto University, vol. 19, p. 11-56.

Steinberg, G.S. and A.S. Steinberg (1975) On possible causes of volcanic tremor. *Journal of Geophysical Research*, vol. 80, p 1600–1604.

St. Lawrence, W. and A. Qamar (1979a) Transient water flow in subglacial channels. *Journal of Glaciology*, vol. 23, p. 432–433.

St. Lawrence, W. and A. Qamar (1979b) Hydraulic transients: A seismic source in volcanoes and glaciers. *Science*, vol. 203, p. 654–656.

Tanaka, Y. (1979) On the type of volcanic earthquakes at Asamayama volcano. *Papers in Meteorology and Geophysics*, vol. 30, p. 61–74.

Wylie, E.B. and V.L. Streeter (1978) *Fluid Transients:* New York: McGraw-Hill.

Zielke, W. and H.P. Hack (1972) Resonance frequencies and associated mode shapes of pressurized piping systems. *International Conference on Pressure Surges* (British Hydraulic Research Association), p. G1-1–G1-13.

A facsimile catalog card in Library of Congress MARC
format is reproduced below.

END

FILMED

2-83

DTIC